Complex-Variable Vector Magnetic Characteristic Analysis considering Residual Stress Effect

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Abstract—This paper presents a modeling of vector magnetic properties considering residual stress effects in electrical steel sheets. Because the practical phenomenon is too complicated, complex variable expressions in linearization of magneto-elastic effects are applied to the modeling. We call this SCES (Stress-Complex E&S Modeling). The validity of the developed modeling was verified in comparison with the measurement. The result shows that the developed method is very effective to reduce necessary conditions in the approximate expression of the material coefficients depending on stress.

Index Terms—Hysteresis modeling, vector magnetic property, residual stress, complex variable, finite element method.

I. INTRODUCTION

We have so far developed high efficiency motors considering detailed vector magnetic properties such as the rotational iron loss distributions with the developed E&S modeling [1]. However, the magnetic power loss in the constructed prototype machine increased over the estimated one in the designing. It is therefore necessary to consider effect of residual stress in core materials in designing of rotating machines.

We have developed an evaluation method of the vector magnetic property under tensile and compressive stress [4]. In this paper, we report a developed numerical modeling of vector magnetic properties under the residual stress by using the previously developed database. In this modeling we used approximated formulation with complex variables so called the complex-variable E&S (CES) modeling [5] for simplification. In verification of the developed modeling, the iron loss depending on the residual stress in core materials in designing of rotating machines.

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II. NEW NUMERICAL MODELING

A. Stress-Complex variable E&S Modeling

Fig. 1 shows parameters used in definition of CES modeling. Here $B_{\text{max}}$ is the maximum magnetic flux density, $\theta_B$ the angle between the rolling direction (R.D.) of core material and direction of $B_{\text{max}}$ (Inclination angle), $\sigma$ the ratio of the minimum magnetic flux density and $B_{\text{max}}$ (Axis ratio), $\sigma_1$ the maximum principle stresses, $\sigma_2$ the minimum principle stresses and $\theta_B$ the angle between the R.D. of core material and direction of $\sigma_1$.

From the above assumptions and expanding the CES modeling [5], the SCES modeling can be written by

$$\hat{H}_k = \bar{\nu}_x \left( B_{\text{max}}, \theta_B, \sigma_1, \sigma_2, \theta_B \right) \hat{B}_x + j\omega \bar{\nu}_y \left( B_{\text{max}}, \theta_B, \sigma_1, \sigma_2, \theta_B \right) \hat{B}_y$$

where $B_k$ is the components of the magnetic flux density, $\nu_x$ the magnetic reluctivity coefficients, and $\nu_y$ the magnetic hysteresis coefficients. The variables with upper point mean complex variables and the over bar means effective variables. $k$ indicates the component of $x$ or $y$, and $\omega$ is the angular frequency.

B. Approximation of the Stress Conditions

The coefficients $\nu_x$ and $\nu_y$ are derived from data measured with a vector magnetic property measurement system, which can apply any stress up to 50 MPa to a cross-shaped sample [4]. However, because there are many parameters ($B_{\text{max}}, \sigma_1, \sigma_2, \theta_B$), a great number of measurement conditions exist and it is impossible to measure the magnetic properties depending on stress for all the conditions within one year. In order to reduce the necessary data in the modeling, we propose three approximation methods in this paper.

(i) Method-I

Based on the measured iron loss properties versus the principle stresses, $\sigma_1$ and $\sigma_2$, we discovered a simple relationship between the magnetic properties and the principle stresses. Fig. 2 shows the magnetic power loss $W_x$ depending on the principle stresses under alternating flux condition ($B_{\text{max}} = 1T$), where, the negative stress means compressive stress and the positive stress means tensile stress. The conditions of Fig. 2 (a) are $\theta_B = 0$ deg. and $\theta_B = 0$ deg., and those of Fig. 2 (b) are $\theta_B = 90$ deg. and $\theta_B = 0$ deg. From these figures, we can assume that (2) is satisfied. Because the maximum flux density is controlled to be 1T during the measurement, we can assume that the field strength (2) and the hysteresis loop are almost same when the iron losses are nearly equal although the principle stresses, $\sigma_1$ and $\sigma_2$ are different.
The magnetic properties when \( \theta = 0 \) deg., can be approximated with a rotation formula. To express properties in the other directions, we have to modify the magnetic property in the rolling direction to be a little bit too good. Hence we propose to use the measured results in the rolling direction to be a suitable one for example and rotate with the method-II. The conditions can be derived by adding a correction stress value until the field strength trajectories (the maximum field strength) agree well. We call this correction stress "relative stress".

**Fig. 2.** The magnetic power loss at each stress in x- and y-direction \( (\theta_{\text{ave}} = 1T, \alpha = 0) \).

**Fig. 3.** Definition of angles used in simulations.

\[
H_j(\sigma_x, 0) = H_j(\sigma_x + \sigma_y, \sigma_y) \quad (j = \sigma_x, \sigma_y)
\]  

For example, the similar \( W_m \) was observed when \( (\sigma_x, \sigma_y) = (0, 0), (\sigma_x, \sigma_y) = (10, 10), \) and \( (\sigma_x, \sigma_y) = (-10, -10) \). From the results as shown in Fig. 2, the magnetic properties for the conditions on the broken lines as shown in Fig. 3 are similar and they can be approximately obtained from the magnetic properties under the conditions of the gray-dotted points on the \( \sigma_x \)-axis.

(ii) Method-II

Because the direction of the principal stress \( \sigma_1 \) is changeable, it is very important to know the magnetic properties depending on \( \theta_s \). However the number of the measurement conditions becomes enormous and data become huge. Hence we propose to use the measured results in the R.D. under any \( \sigma_x \) \( (\theta_s = 0 \) deg.\) instead of \( \sigma_1 \) for all the conditions. The magnetic properties when \( \theta_s \) does not equal 0 deg. can be approximated with a rotation formula.

(iii) Method-III

The simple coordinate transformation is not enough because the sheet material has magnetic anisotropy even non-oriented steel sheet, therefore we have to change the magnetic properties depending on the exciting direction \( \theta_s \). Because the magnetic property in the rolling direction is little bit too good to express properties in the other directions, we have to modify the magnetic property in the rolling direction to be worse by adding a compressible stress to be a suitable one for example and rotate with the method-II. The conditions can be derived by adding a correction stress value until the field strength trajectories (the maximum field strength) agree well. We call this correction stress "relative stress".

**Fig. 4.** Comparison of the iron losses calculated by using the approximated method (Right) with the measured ones (Left).

### III. RESULTS AND DISCUSSIONS

Fig. 4 shows comparison of the iron losses depending on \( \theta_s \) calculated by using the approximated method (Right figure) with the measured ones (Left figure). As shown in this figure, the calculated results were agreed well with the measured ones and we could confirm the validity of the SCES modeling. For a more practical application, we analyzed magnetic characteristic distributions in a ring core model with the SCES modeling. The detailed results will be shown in the full paper.

### IV. CONCLUSION

In this paper, we have reported the new numerical modeling of the vector magnetic properties under residual stress. In order to reduce measurement conditions and number of parameters in the fundamental SCES modeling, we have proposed three approximation methods on the stress conditions. As a result, we could successfully reduce necessary conditions in the approximate expression of the material coefficients depending on stress. The SCES modeling was verified in the analysis of the magnetic power loss in any direction. Applications of the modeling for more practical model cores will be shown in the full paper.

### REFERENCES


