FEM-BEM Analysis of Radio Frequency Drying of a Moving Wood Piece

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Abstract—The electric field has been computed using the hybrid technique FEM-BEM, so the motion modifies only the boundary conditions on the surface of the wood piece; FEM mesh inside the piece as well as some of the elements of the matrix associated to the inner nodes remain unchanged. The electric field problem is coupled with the thermal field problem because the complex permittivity depends on the temperature and the specific losses are influenced by the electric field strength. Besides that, the moisturized wood to be dried is moving and the water vaporization has impact on the thermal field. Our paper proposes a method for solving the electric field problem coupled with thermal diffusion and mass problems, taking into consideration the movement of the wood object exposed to the drying process.

Index Terms—Finite element methods, dielectric losses, wood industry, frequency, voltage.

I. INTRODUCTION

The electric field inside the drying oven is produced by a set of electrodes powered by high voltages (8-20kV) and frequencies of: 13.56, 27.12 and 40.68 MHz. The heating of the objects by radio frequency electromagnetic field produces a volume distribution of the specific losses that leads to a uniformly enough level of distributed thermal field inside the moistened piece to be dried. The wavelength (>10m) is greater than the oven dimensions; there are not ferromagnetic parts, thus the derivative of magnetic flux density from Faraday’s law can be neglected. A quasi-electrostatic problem has to be solved, where the electric permittivity is a complex quantity that depends on the temperature, therefore the electric field problem is coupled with the thermal diffusion one. The specific losses in the thermal diffusion equation depend on the electric field strength and the complex permittivity. The motion of the wet wood changes the geometrical structure needed for solving the electric field problem. Therefore the hybrid technique FEM-BEM is recommended for the computation of the electric field. This method presents the main advantages of the FEM [1], [2]. The solution of the electric field problem coupled with the thermal and the mass ones for a 2D structure was presented in [3].

Our paper proposes an analyzing procedure of the drying process for three-dimensional pieces of wood. The integral equation on the 2D boundaries of the piece and of the electrodes defines the stiffness matrix that is the boundary condition for the electric field problem analyzed with FEM inside the piece.

The mathematical model of water vaporization is particularly complicated if we consider the diffusion of the water and the vapors within the volume of the wood related to its fibrous irregular structure. Moreover, this structure depends on the piece to be dried and it cannot be generally known. Therefore, we may assume that there is only surface vaporization, and then the diffusion inside the wood piece is very rapidly done. Besides, the vaporization inside the piece must be avoided since it may lead to unwanted cracks. Evaporation on the wood surface reduces the temperature and it is part of the boundary condition of the thermal field problem. The speed of the wood piece inside the oven has to be determined, such that the imposed moisture level to be reached at the exit from the oven.

II. FEM DISCRETIZATION OF THE ELECTRIC FIELD PROBLEM INSIDE THE PIECE

The solution of the sinusoidal electric field problem in the wood domain $\Omega$ is obtained by using phasor representation. The complex permittivity of the wood is: \( \varepsilon = \varepsilon' - j\varepsilon'' \). Since we can neglect the derivative of magnetic flux density in the Faraday law, the electric potential satisfies equation:

\[ \nabla \cdot \varepsilon \nabla \varphi = 0 \]  \hspace{1cm} (1)

The boundary condition:

\[ \frac{\partial \varphi}{\partial n} = L(\varphi) + U \]  \hspace{1cm} (2)

is given by the integral equation written on the boundary $\partial \Omega$ (see par.III). $U$ is the values of $E$-normal component produced by the electrodes when $V=0$ on $\partial \Omega$ and $L(\varphi)$ is produced by the boundary potential when the electrodes potential is zero. The Galerkin discretization of equation (1) is:

\[ \int_{\Omega} \nabla \varphi \cdot \nabla \varphi dV + \int_{\partial \Omega} \varphi \varepsilon \frac{\partial \varphi}{\partial n} d\bar{l} = 0 \]  \hspace{1cm} (3)

where $\varphi$ is the shape function.

III. THE INTEGRAL EQUATION OF THE ELECTRICAL POTENTIAL IN BEM

On the boundary $\partial \Omega_0$ of the air domain $\Omega_0$, consisting of the electrodes surfaces $\partial \Omega_k$, $k > 1$ and the boundary $\partial \Omega$ of the wood, the following integral equation is valid [4]:

\[ \varphi(P) = \int_{\partial \Omega} \frac{R}{R^3} \varphi dS_Q - \int_{\partial \Omega} \frac{\partial \varphi}{\partial n} \frac{1}{R} dS_Q - \sum_{k} \int_{\partial \Omega_k} \frac{\partial \varphi}{\partial n_k} \frac{1}{R} dS_Q \]  \hspace{1cm} (4)

where: $\theta$ is the solid angle under which a small vicinity of the domain $\Omega_0$ is seen from the observation point $P$, and $n$ is the
inner normal unit vector, at the integration point \( Q \). If the observation point \( P \) is placed on \( \partial \Omega_k \), then the left side of relation (1) becomes \( 4\pi V_k = 0 \).

We approximate the boundary \( \partial \Omega \) by a polyhedral surface with triangular sides and we hypothesize that the derivative with respect to the normal unit vector \( \partial V / \partial n \) on each side to be constant, while \( V \) has a linear variation defined by the values of the potential in the nodes that define the triangle. If we integrate the relation (4) on the triangular sides of the boundaries \( \partial \Omega \) and \( \partial \Omega_k \), we obtain the relation (2) that represents the boundary condition for FEM inner problem.

IV. THE THERMAL DIFFUSION PROBLEM, COUPLED WITH THE MASS PROBLEM AND MOTION OF THE WOOD PIECE

The diffusion of the thermal field is described by the equation:

\[
-\nabla \lambda \nabla T + c \frac{\partial T}{\partial t} = p
\]

(5)

where \( \lambda \) is the thermal conductivity, \( c \) is the volume thermal capacity, and the specific losses in the dielectric are given by relation: \( p = E^2 \varepsilon \omega \varepsilon' \varepsilon_0 \delta \) where the electric field strength \( E \) is obtained from the electric field problem. The boundary condition is:

\[
-\lambda \left( \frac{\partial T}{\partial n} \right) = \alpha \left( T - T_e \right)
\]

(6)

where: \( \alpha \) is the coefficient of thermal transfer on the surface, and \( T_e \) is the temperature outside the piece of wood. The numerical solution of the equation (5) is given by FEM, using the same mesh as within the problem of electric field, while the time discretization has been done using the trapezoidal rule.

The vaporization of the water from the wood mass takes place in small part inside the piece of wood and largely on its surface. To take into consideration the inner vaporization leads to the computation of a complicated water diffusion problem in which a non-homogeneous pressure field interferences due to the water vapors. The high anisotropy of the wood, due to the orientation of the wooden fibers, makes almost impossible the water diffusion problem to be accurately modeled. Additionally, for drying processes, the rapid appearance of water vapors from the inner part of the wood can lead to its destruction. For this reason, the maximum temperature inside the wood object has to be limited (below 70°C). Thus, we can neglect the inner vaporization and take into consideration only that one on the surface of the wood. The evaporation speed on the surface unit depends on the difference between the temperature on the surface of the wood and the ambient temperature. It also depends on the degree of saturation of the vapors, on the air pressure, on the air flow in the proximity of the wood object etc.

We admit that the evaporation speed \( \frac{d \tau_s}{dt} \) of the water on the unit surface linearly depends on temperature:

\[
\frac{d \tau_s}{dt} = w(T - T_c)
\]

(7)

If \( A \) is the latent heat of the vaporization volume, then the loss of the heat due to the vaporization on the surface reduces the temperature on the surface alike the thermal convection. So, we can take into consideration the vaporization by using a virtual convection coefficient in the boundary condition (6), according to the relation:

\[
\alpha_{vc} = \alpha + \alpha v
\]

(8)

The estimation of the temperature field on the interval \( [t_i, t_{i+1}] \) allows the calculation of the water volume evaporated during this interval:

\[
-(V_{water}^i - V_{water}^{i+1}) = \frac{1}{2} \int_{\partial \Omega} w(T(t_{i+1}) + T(t_i)) \Delta t_i dl
\]

(9)

This leads to a change of the piece moisture. The physical parameters of the wood piece depend on temperature and moisture and they are iteratively rectified at each time step.

The computation of the moving speed inside the oven requires the following procedure: the requisite time for reaching the imposed moisture value for the stationary piece is calculated; then the oven active length is divided by this time. Since the electric field depends on the position, the speed can be slightly reduced.

V. CONCLUSIONS

The computation of thermal field change for wood pieces dried by radiofrequency described by our paper can be particularly useful in order to control the maximum values of the temperature and to estimate the drying time and the moving velocity of the piece. The extended paper will present computation details and an illustrative example.

REFERENCES