Abstract—This work presents an efficient procedure for evaluating singular integrals arising in the Method of Moment analysis of arbitrary wire structures. The singular kernel is modeled as an elliptical integral of the first kind and evaluated using the Arithmetic Geometric Mean. Numerical integration is applied only for no singular kernels. It is shown that the efficient singularity removal process reduces the condition number of the applied only for no singular kernels. It is shown that the efficient singularity removal process reduces the condition number of the applied only for no singular kernels. It is shown that the efficient singularity removal process reduces the condition number of the singularities treatment [5].

Numerical results considerable precautions should be taken in the singularities treatment [5].

In the MoM technique, the choice of base functions, for the representation of the equivalent current, and weight functions is very important for the accuracy and convergence of the numerical analysis. However, sophisticated base or weight functions may lead to complicated integrands and, consequently, singularity removals. To obtain accurate numerical results considerable precautions should be taken in the singularities treatment [5].

In the present work a robust numerical technique is proposed to remove the singularities arising in the EFIE associated with electromagnetic scattering by arbitrary wire structures. For MoM analysis triangular base and weight functions (TF) are used. The TF, normally, ensures a good representation of current physical behavior and provides relatively simple integrands in the MoM analysis. In the present work the TF is defined over two consecutive segments, as illustrated in Fig. 1. The Galerkin method is used in the MoM analysis and, consequently, the weight functions are defined as well. The technique proposed for singularity removal models the singular kernel as an elliptical integral of the first kind and evaluates it using the Arithmetic Geometric Mean (AGM). Gaussian quadrature is applied only to evaluate integrals with no singular kernels.

The EFIE for arbitrary wire structures is [6]:

\[ \hat{n} \times \mathbf{E}'(r') = \hat{n} \times \int_{\partial \alpha} \left[ k^2 \mathbf{I}(t') G(r - r') - \nabla \mathbf{I}(t') \nabla G(r - r') \right] dt', \]  

(1)

where \( \hat{n} \) is the unit vector normal to the wire direction, \( t, k \) is the wave number, \( \mathbf{E}' \) is tangential component of the electric incident field, \( \mathbf{I} \) is the surface current in the wire, \( \mathbf{G} \) is the electric permittivity of the medium, \( \mathbf{r} \) and \( \mathbf{r}' \) represent the observation and source points, respectively and \( G(r - r') \) is the Green free-space function.

The MoM solution of (1) using TF as base and weight functions, leads to integrals in the form:

\[ I = \int_{a=1}^{1} \int_{a'=1}^{1} ab \left( e^{-ikR}/R \right) da' da, \]  

(2)

where \( R \) is the distance between source and observation points, \( a \) and \( b \) may be equal to 1, \( a \) or \( a' \). The integrals in (2) have removable singularities whenever the observation point is very close to the source point, i.e., when \( R \to 0 \).

The technique used in this work to treat these singularities is redefine (2) as an average around the circumference of the source and observation segments.

II. INTEGRAL EQUATION EVALUATION

Fig. 1. Distribution of triangular functions
\[
I = \frac{1}{2\pi} \int_{\gamma} \left( \int_{\gamma'} \frac{1}{R_i} a e^{i\theta_{ij}} da' da \right) d\gamma d\gamma', \quad (3)
\]

where \(\gamma\) and \(\gamma'\) are the angles around the circumference of the source and observation segments and \(R_i\) is

\[
R_i^2 = R_i^2 + (a + a')^2 - 4aa' \cos[(\gamma - \gamma')/2]. \quad (4)
\]

Considering the change of variable \((\gamma - \gamma'/2)\rightarrow(\varphi - \pi/2)\)

(3) can be separated into two parts and rewritten as:

\[
I = \frac{1}{2\pi} \int_{\gamma} \frac{2^2}{\sqrt{1 - \sigma^2}} \sin^2 \phi \varphi + 1 \frac{1}{8\pi^2} \int_{\gamma} e^{-\sigma R} - 1 d(\gamma' - \gamma'). \quad (5)
\]

where \(R_i = \sqrt{R_i^2 + (a + a')^2}\).

The second term of (5) does not have singularities and is numerically evaluated by Gaussian quadrature. The first term of (5) has the form of a first kind elliptical integral and presents singularities. In this work this term is evaluated using Arithmic Geometric Mean according to the algorithm illustrated in Fig. 2 [7]. This procedure leads to highly accurate results even for \(\sigma \rightarrow 1\) (Singularity).

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{agm_algorithm.png}
  \caption{AGM algorithm [7]}
\end{figure}

III. NUMERICAL RESULTS

To evaluate the accuracy of the technique proposed we consider a dipole with a radius of 0.01 \(\lambda_0\) and a length of 0.47 \(\lambda_0\), where \(\lambda_0\) is the vacuum wavelength. The excitation is performed using the Delta Function Excitation Method [6].

The accuracy of the numerical results was verified against the result obtained by software 4NEC2 using the following Mean Relative Error:

\[
E_{\text{MR}} = \frac{100}{N} \sum_{i=1}^{N} \left[ \frac{X^{\text{MoM}}(i) - X^{\text{4NEC2}}(i)}{\max(X^{\text{4NEC2}}(i))} \right] \%,
\]

where \(X\) represents the results obtained by MoM or 4NEC2 and \(N\) is the number of segments used to describe the dipole.

In the analysis \(N\) varied from 9 to 41, the \(E_{\text{MR}}\) obtained, for current distribution, is shown in Fig. 3 and the condition number in Fig. 4. In both analyses we consider the situation where the technique to remove the singularities is and is not applied. As can be verified, when this technique is applied the number of base functions required to represent the equivalent current can be reduced once the \(E_{\text{MR}}\) is always less, independent of the number of segments used. It can also be observed that when the extraction technique is used, the MoM matrix is better conditioned, especially when the number of segments employed is greater.

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{condition_number.png}
  \caption{\(E_{\text{MR}}\) with and without singularity removal}
\end{figure}

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{condition_number.png}
  \caption{Condition number with and without singularity removal}
\end{figure}

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REFERENCES


