Vector Design Optimizations Using an Improved Cross-Entropy Method

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Abstract—An improved vector cross-entropy method is proposed to solve multi-objective design optimizations of inverse problems. To balance exploitation and exploration searches, the entire evolutionary process is recast to two phases: diversification and intensification phases. Different parameter updating mechanisms of the probability density function (pdf) are designed for each phase. To enhance the diversity of populations, several different pdfs are evolved synchronously; to guarantee uniform distribution of the searched Pareto solutions, an elite projection mechanism is proposed. Mathematical function and an inverse problem are used to testify the proposed method has also been extended to multi-objective optimizations for linear antenna arrays.

I. AN IMPROVED CROSS-ENTROPY METHOD

A. Cross-Entropy method

Based on estimating the probabilities of rare events, the Cross-Entropy (CE) method is firstly proposed as an effective single objective optimizer [1]. Due to the simplicity in both mathematical concepts and numerical implementations, the CE method has also been extended to multi-objective optimizations [2]-[3]. However, a large number of sampling points are still needed to precisely evolve the parameters of pdfs. To speed up the convergence speed and obtain better distribution of the Pareto front, some improvements are proposed.

B. The Diversification Phase

To balance the exploitation and exploration searches, the whole iterative procedure is divided into two phases: diversification and intensification phases. Diversification phase emphasizes on guaranteeing the diversity of sampling points and intensification phase aims on ensuring a fast convergence. Therefore, different parameter evolutionary mechanisms are proposed for pdfs in each phase. The algorithm starts from a diversification phase to uniformly explore the whole feasible space; because of simplicity to implement, the normal distribution function $N(\mu, \sigma^2)$, with its mean $\mu$ and standard deviation $\sigma$, is selected as the pdf. To enhance the diversity of sampling points, several different pdfs are evolved synchronously. In the diversification phase, the parameters of pdfs evolve using:

$$\mu_j(t+1) = S_j(t)$$  \hspace{1cm} (1)

$$\sigma_j(t+1) = \text{std}(x_j^{\text{new}}(t))$$  \hspace{1cm} (2)

$$\sigma_j(t+1) = a\sigma_j(t+1) + (1-a)\sigma_j(t)$$  \hspace{1cm} (3)

$$\alpha = \sigma_0 - \sigma_t(1 - 1/t)^q$$  \hspace{1cm} (4)

where $j$ is the index for the $f^{th}$pdf; $S(t)$ is a set of fixed length, comprised of members of mean($x_j^{\text{new}}(t)$) and external archive $A$ based on tournaments; $\rho$ is the elite rate of one sampling; $q$ is an attenuation factor, which is decided by the designer.

C. The Intensification Phase

The intensification phase is designed to efficiently and precisely locate the Pareto solutions. In the intensification phase, the parameters of pdfs are evolved as follows:

$$\mu(t+1) = A(t_{\text{elite-related}})$$  \hspace{1cm} (5)

$$\sigma_j(t+1) = \text{std}(x_j^{\text{new}}(t))$$  \hspace{1cm} (6)

$$\sigma_j(t+1) = \beta\sigma_j(t+1) + (1-\beta)\sigma_j(t)$$  \hspace{1cm} (7)

where $A(t_{\text{elite-related}})$ is the corresponding elite component in the external archive; $\rho$ is the elite rate of one sampling; $\beta$ is a smoothing parameter, which is in the region of 0.6–0.8.

D. Elite Projection Mechanism

In order to guarantee the uniform distribution of the searched Pareto solutions, an elite projection mechanism is proposed as: Project the elite individuals to the utopia sub-domains; classify the projected points into the corresponding sub-domains and calculate the distance between the projection points and the centers of its corresponding sub-domains. Compared with multi-objective normal boundary intersection method, only elite individuals are projected onto the utopia plane in the proposed CE, which decreases the computational burden effectively.

E. Tuning of the Initial Parameters of pdfs

For the proposed CE method, two parameters, the initial location of parameters of pdfs and the sampling size, should be tuned carefully. If the initial values of these parameters are too close to the boundary of the searching space, a lot of sampling points will exceed the boundary; and if the initial parameters are too close to each other, the exploration ability is degraded. As a result, a large amount of sampling points are needed to modify the searching direction for the two cases. Since the initial parameters are selected randomly in existing CE methods, the aforementioned two ultimate cases may occur. To address this issue, a novel method to determine the initial parameters of the pdfs is proposed as: generate $N_0$ sampling points randomly; and classify the sampling points into the corresponding sub-domains according to the projective distance; after sorting the sampling points based on
sub-domains and dividing them into some fixed-length matrix, a series of \( \mu_D \) and \( \sigma_D \) are finally calculated from each matrix.

**F. Algorithm Description**

To facilitate the understanding of the proposed method, the details about its iterative process are summarized as follows:

Step 1: Calculate the utopia plane.
Step 2: Divide the utopia plane into sub-domains.
Step 3: Calculate the initial parameters of pdfs.
Step 4: Define \( N_D \) the sampling number of the diversification phase; \( N_I \) the sampling number of the intensification phase; \( N_P \) the population size for one pdf family sampling; \( A \) the external archive; \( E \) the set of best solutions in one iteration. Initialize iterative number \( r=0 \); Generate the initial population \( P \) using the normal distribution function with \( \mu_D \) and \( \sigma_D \).

Step 5: Calculate the function values of population \( P \) and compute the \( E \) using a fast non-dominated sorting approach [4]. Project the individuals of \( E \) into the utopia sub-domains. Classify the projected points into the corresponding sub-domains and calculate the distance between the projection points and the centers of their corresponding sub-domains. Update external archive \( A \) with individuals of the minimum distance and the lowest rank in the same sub-domains both in set \( A \) and \( E \).

Step 6: If \( t \leq N_D \), \( \mu_D \) is updated based on the tournament mechanism from set \( A \) and \( \mu_I \) updated from elite points of each sampling population; \( \sigma_D \) is updated based on the elite points of each sampling population, and then \( \sigma_I \) is smoothed using a dynamic function. If \( N_D < t \leq (N_D + N_I) \), \( \mu_D \) is updated using the external archive \( A \); \( \sigma_D \) is updated based on the elite points of each sampling population. \( \sigma_I \) is smoothed using a constant function. If \( t > (N_D + N_I) \), go to Step 7.

Step 7: Stop the algorithm.

**II. Numerical Results**

To validate and demonstrate the advantages of the proposed algorithm, a series of test functions (MOP4) and the linear antenna array optimization problem [5] are solved.

The parameters of the proposed algorithm for solving MOP4 are set as: \( N_D = 20 \), \( \rho = 10\% \), \( N_D = 10 \), \( N_I = 20 \), \( q = 50 \), \( \beta = 0.7 \); Fig. 1 gives the searched Pareto front using the proposed method. Table I tabulates the comparisons of the original MOCE [3], NSGA2 and the proposed method.

![Fig. 1 The searched Pareto front of MOP4 using proposed method](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Min ( F_1 )</th>
<th>Min ( F_2 )</th>
<th>No. of Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original MOCE</td>
<td>-20.0000</td>
<td>-11.6273</td>
<td>1500000</td>
</tr>
<tr>
<td>NSGA2</td>
<td>-19.9974</td>
<td>-11.6273</td>
<td>28000</td>
</tr>
<tr>
<td>The proposed</td>
<td>-20.0000</td>
<td>-11.6271</td>
<td>20418</td>
</tr>
</tbody>
</table>

The algorithm parameters for solving the linear antenna array optimization are set as: \( N_D = 50 \), \( \rho = 10\% \), \( N_D = 15 \), \( N_I = 20 \), \( q = 50 \), \( \beta = 0.7 \) to update the parameters of pdfs using (1)-(7). Fig. 2 presents the searched Pareto front using the proposed method, and Table II gives the comparison on the final solutions of well designed evolutionary algorithms and the proposed method using the benchmark value of a uniform antenna array. Obviously, the proposed algorithm can find the same qualified solution but using a relative small number of iterations.

**III. References**


