

Harmonic pressure optimization on numerical electric motor model

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Audible noise of electric machines has become an important criterion in their design. In this paper we used a multi-physics numerical model (electromagnetic-dynamic) in order to predict mechanical vibration caused by magnetic pressure in a wound rotor synchronous machine. The final objective is to reduce this mechanical vibration. The novelty of this paper is to use an optimization method with this multi-physics numerical model in order to reduce important vibration peaks thanks to magnetic pressure harmonics.

Index Terms—Electric machines, frequency response, harmonic analysis, magnetic forces, numerical models, optimization, vibrations.

I. INTRODUCTION

As a result of increasing usage of electric machine especially in vehicles, and with the respect to the European standards, it has become necessary to reduce noise damages emitted by such machines, while keeping a good proportion of benefits. In our study we optimized the motor on magnetic pressure criteria, unlike most cases where optimization focus on loss energy or torque level and quality [1]-[2]. Analytical models have been developed in order to calculate magnetic pressure and vibrational behavior [3]. In this context, optimizations were done on discrete criteria (slot numbers) [4]. A new approach is to optimize the spectrum of the magnetic pressure, then by applying a transfer function enable us to observe their vibratory effect.

Two finite element models are modeled:

- 2D Electromagnetic model to calculate magnetic pressure applied on the stator.
- 3D Mechanical model to calculate the frequency response.

A coupling tool implemented by MATLAB project magnetic pressure using Flux 2D [5] on the mechanic model, and calculates frequency response using Nastran. With good precision and acceptable computation time, this coupling is seems to be the best way to calculate magnetic vibration.[6]-[7]

represents the ¼ of the machine with 7062 T3 elements (3-noded triangle), unlike the dynamic 3D model who represent the whole stator with 26400 H8 elements (8-noded hexahedron). The magnetic model is finer then the dynamic model, in order to properly calculate the magnetic pressure harmonics. (Fig 1)

B. Magnetic pressure

The magnetic pressure, also known as Maxwell pressure is applied to the interface between two magnetic materials of different permeability. In the electrical machines, this pressure appears clearly at the interface between the air gap and stator inner surface. The magnetic pressure along the air gap can be expressed [8]:

\[
\sigma_n = \frac{1}{2\mu_0} \left( [B_n(\theta, t)]^2 - [B_t(\theta, t)]^2 \right)
\]

(1)

Where \( \theta \) and \( t \) represent angular position and time respectively, \( \mu_0 \), \( B_n \) and \( B_t \) are the permeability of air, the radial and tangential flux density in the air gap. Since that the material of the stator has a much higher permeability compared to the air, we consider at the beginning that tangential flux is negligible. Hence the magnetic pressure can be written as:

\[
\sigma_n = \frac{1}{2\mu_0} [B_n(\theta, t)]^2
\]

(2)

We can see in Fig. 2 magnetic pressure in function of position (\( \theta [0 : 90^\circ] \)) and time (one electric period); calculated with 2D electromagnetic finite element solver (Flux 2D).

Fig. 1. Wound rotor synchronous machine

Fig. 2. Magnetic pressure
C. Frequency response

While rotor rotates, the magnetic pressure apply on stator varies in function of time (Fig. 2) making the stator vibrate. This vibration can cause a lot of problem (fatigue, acoustic ...), especially if we take into consideration the mechanical resonance. This magnetic pressure calculated is projected onto a mechanical mesh of the stator thanks to a multi-physics coupling tool implanted in Matlab, in order to calculate the frequency response (MSC. Nastran).

\[ M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = f(t) \quad (3) \]

Forced vibrations are described by (3); \( M, C \) and \( K \) are respectively the global mass matrix, damping and stiffness, \( f \) is the load vector and \( u \) is the displacement vector.

In our case, we use the harmonic loading where \( \omega \) is the pulsation:

\[ f = f(\omega)e^{j\omega t} \quad (4) \]

\[ u(t) = u(\omega)e^{(j\omega t - \varphi)} \quad (5) \]

By substituting (4) and (5) in the (3) we obtain:

\[ -\omega^2 Mu(\omega) + j\omega Cu(\omega) + Ku(\omega) = f(\omega) \quad (6) \]

So as to reduce the size of the problem and obtain a decoupled system of equation, we used the modal superposition method. We consider a truncated base \( \Phi \) which does not contain all the modes in the base \( \Phi \). The size of this truncated base must be about 2 to 3 times the interested frequency range. [9]

\[ u(\omega) = \sum_{i=1}^{m} \phi_i q_i(\omega) = \Phi q(\omega) \quad (7) \]

This multi-physics model was validated experimentally by vibration tests on similar prototypes machines. An order tracking (Vibration harmonic 44) for different motor speed is presented in Fig. 3. (for confidential reason Acceleration is presented on relative scale).

III. OPTIMIZATION METHOD

A. Optimization methods

Optimization consists to determine the best element of a set of design parameters, in the sense of a given quantitative criterion. It exist two major methods of optimization, the deterministic and the stochastic methods. The first group consists to determine a direction or trend of the function. They are economical but can be trapped in a local optimum. The second group explores the domain based on large number of random trial. They are efficient to find the global optimum but are very costly.

B. Numerical design of experiment

In order to reduce the number of numerical computation it is recommended to construct a response surface. This analytical function, which all their characteristics are known, is built by interpolation of the original function based on selected experiment points (Fig. 3). For exemple, \( (8) \) is a quadratic model of approximation with design parameters interaction.[10][11][12]

\[ F'(x) = a_0 + a_1 x_1 + a_2 x_2 + a_{12} x_1^2 + a_{22} x_2^2 + a_3 x_1 x_2 \quad (8) \]

C. Optimization algorithm

GOT-it is an optimization tool related to Flux2D, it can apply optimization algorithm on electromagnetic numerical model. Since we have a mono objective problem, it is recommended to apply the sequeliet surrogate optimizer (SSO). As you can see in Fig. 4. SSO build a response surface, then apply genetic algorithm (GA) to it \((x_{opt} \text{ is the design parameters})\), then compare the value of the surrogate \( Sf(x) \) to the real one \( f(x) \), if the results are too different, the SSO automatically reiterates the process by refining the response surface by reducing the research domain around the considered point. It is a very efficient optimization tool for such cases. [13]
IV. COMPUTATION

A. Design parameters:

In order reduce the number of optimization computation and according to their vibratory effect we chose to optimize harmonics 4, 12, and 48 (400, 1200 and 4800 Hz respectively) (Fig. 6). Screening study shows that the parameter of rotor arc PA, the slot opening SO and the slot corner radius RS are the most influent on optimization criterions. According to design and manufacturing conditions, the margins of variation are ±30% for PA, ±20% for SO and ±5% for RS (Fig. 5.). The normalized coefficient of influence of each parameter on the different criterion is shown in the table I, each line in this table read separately.

It’s obviously that the efficient parameter and their coefficient are largely different for different harmonic.

Table I: screening results

<table>
<thead>
<tr>
<th>Design parameter</th>
<th>PA</th>
<th>SO</th>
<th>RS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque level</td>
<td>1</td>
<td>0.2</td>
<td>0.03</td>
</tr>
<tr>
<td>Harmonic 4</td>
<td>1</td>
<td>0.18</td>
<td>0</td>
</tr>
<tr>
<td>Harmonic 12</td>
<td>1</td>
<td>0.42</td>
<td>0.15</td>
</tr>
<tr>
<td>Harmonic 48</td>
<td>1</td>
<td>0.06</td>
<td>0</td>
</tr>
</tbody>
</table>

B. Initial values

You can see in Fig. 6 the spectrum of the curve of magnetic pressure calculated on a point situated on the inner surface of the stator. We load the mechanical model by magnetic pressure, using the coupling tool, and calculate the frequency response Fig. 7. We can localize two important peaks at 6 and 7.2 kHz, others less important around 2.4, 4.8, 8 and 9 kHz.

C. Torque optimization

Results with classical optimization show (Fig. 8) that we can increase the torque about 3.5% while maintain the peak in his level, by changing the rotor arc parameter PA to his maximum and the slot opening SO to his minimum.

In other side we can decrease the peak to peak amplitude about 16.8% if we decrease PA about 15% and decrease SO to his minimum.

D. Harmonics optimization

1. Harmonics optimization without loss in torque

In this part an original optimization criterion is tested. The shape of the curve of magnetic pressure (Fig. 6) was optimized by choosing different harmonics as objective function. The Table II shows the different design model, optimized harmonics, optimized values of influential design parameters, and the gain in peak value. According to optimization results, an optimization of a precise harmonic can varies the other harmonics by different level.

Table II: optimization results (0% average torque loss)

<table>
<thead>
<tr>
<th>Model</th>
<th>Principal harmonic optimized</th>
<th>Influential parameter</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>H4 (400 Hz)</td>
<td>PA (-15%) SO (-20%)</td>
<td>-16.8%</td>
</tr>
<tr>
<td>D2</td>
<td>H12 (1200 Hz)</td>
<td>PA (-17%) SO (-20%)</td>
<td>-17.1%</td>
</tr>
<tr>
<td>D3</td>
<td>H 48 (4800 Hz)</td>
<td>PA (-18%) SO (-20%)</td>
<td>-20%</td>
</tr>
</tbody>
</table>

From vibratory level in Fig. 9, it’s clear that peaks at 6 and 9 kHz are largely reduced especially for D2 about 13 dB.
2. **Allow loss in average torque level**

With an accepted loss in torque up to 2%, the peak can be minimized up to 28%. (Table III)

<table>
<thead>
<tr>
<th>Model</th>
<th>Principal harmonic optimized</th>
<th>Influential parameter</th>
<th>Torque</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>D4</td>
<td>H4 (400 Hz)</td>
<td>PA (-28%) SO (-20%)</td>
<td>-1.8%</td>
<td>-26%</td>
</tr>
<tr>
<td>D5</td>
<td>H12 (1200 Hz)</td>
<td>PA (-29%) SO (-20%)</td>
<td>-1.7%</td>
<td>-26.8%</td>
</tr>
<tr>
<td>D6</td>
<td>H 48 (4800 Hz)</td>
<td>PA (-29%) SO (-20%)</td>
<td>-1.8%</td>
<td>-27.8%</td>
</tr>
</tbody>
</table>

From vibratory level in Fig. 10, we reduced peaks at 6 and 9 kHz about 14 dB for D5 and D6. Peak at 2.4 kHz is reduced about 9 dB for D4, and the peak at 7.2 kHz is decreased about 6 dB for all design model.

**Fig. 10. Vibration results (0% average torque loss)**

![Graph showing vibration results](image)

**Table III: optimization results (2% average torque loss)**

**Fig. 11. Vibration results (2% average torque loss)**

![Graph showing vibration results](image)

**V. CONCLUSION**

This study proved the feasibility and the interest of this optimization method even on unusual criteria (magnetic pressure harmonics).

The mechanical vibrations can be reduced by optimizing some magnetic pressure harmonics. With acceptable decreases for the torque level, we can improve vibration behavior of the machine.

This optimization method, and the coupling tool implemented in Matlab, can be used to predict and optimize magnetic vibration on other type of motor, provided that his geometry is not skewed. In other case a 3D electromagnetic model is required.

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**REFERENCES**


