Comparison of Iron Loss Prediction Formulae

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Abstract— The accuracy of a number of iron loss prediction models was studied. Sinusoidally excited Epstein frame data for the M1924G material was used to analyze each model over a reasonably large range of frequency and induction levels. Such a comparison of some of the most recently developed loss prediction models will help develop practical iron loss prediction algorithms for the next generation of computational electromagnetics tools.

Index Terms—Core loss, electric machine, Epstein test, finite element analysis (FEA), iron loss, laminated steel.

I. INTRODUCTION

Iron losses can be a significant fraction of the total loss in many electrical devices. The need to accurately account for them can be seen by considering the fact that many modern devices such as electric motors operate at very high efficiency (>80%, in many cases) levels and optimizing their performance during design necessarily requires accurate modeling of all sources of loss. This objective, however, has not yet been achieved for iron losses.

Iron loss mechanism is a complex phenomenon that is an active area of research. The fundamental physical characteristics of the loss process are not well understood and there is no standard physics based practical computational model for these losses. Because of this, the empirical approach is recognized as the best practical method for predicting iron losses. This approach can be summarized as follows. Generally, iron loss data are obtained from so called Epstein-frame experiments for a particular material over a range of frequency and induction levels. These data are fitted to a loss prediction formula and the unknown parameters of the model are determined based on a data fitting algorithm. Once the parameters of the model are known, the model can be applied in the computational process.

Most computational electromagnetics tools use a single formula to calculate iron loss values for a variety of applications. This is known to be inadequate and some of the reasons for this will be discussed in this review. In order to derive a hybrid approach that can be applied to a broad set of applications within a single computational tool, a number of modern iron loss prediction formulae have been analysed in this review. As such this work should be seen as a preliminary step towards the development of the next generation of accurate, robust and practical models. In practice, Iron losses are computed in both the frequency and time domains. Here, only the frequency domain formulations are considered. Extension of this work to the time domain is in progress. In Section II of this paper, the empirical models that have been studied for this review are presented. For each model, using an identical set of loss values of a M1924G sample, the accuracy of the loss model is computed for a wide frequency and induction range. In Section III, a summary of the findings is given where the model have been compared with respect to their accuracy, frequency range applicability and algorithmic complexity.

II. IRON LOSS PREDICTION MODELS

Most modern iron loss prediction models are based on the idea of separation of loss components; namely, the hysteresis, the eddy current and the so called excess or anomalous loss component. Bertotti [1] proposed a formula for the loss components (1) that has been the basis of most modern loss prediction formulae,

\[ P_{Fe} = K_{hyt}B^2f + K_{eddy}B^2f^2 + K_{exc}B^{1.5}f^{1.5}. \]  

Bertotti’s Equation (1) was based on the original formula by Steinmetz [2] and is known to have limitations due to which many modifications have been developed over the years. Some of these are considered below.

Model A: A lumped anomalous and hysteresis loss model with the hysteresis loss coefficient of \( B \) fixed at two [3].

\[ P_{Fe} = K_{hyst}B^2f + K_{eddy}B^2f^2. \]  

To apply (2), divide by \( B^2f \) and obtain a linear equation in \( f \). The loss data is fitted to the linear equation at each induction level which then determines \( k_{hyst} \) and \( k_{eddy} \) as a function of \( B \). These coefficients are then fitted as a cubic polynomial as shown in Equations (3) and (4), which fixes the model parameters.

\[ k_{eddy} = k_{e0} + k_{e1}B + k_{e2}B^2 + k_{e3}B^3. \]  

\[ k_{hyst} = k_{h0} + k_{h1}B + k_{h2}B^2 + k_{h3}B^3. \]  

The error between the data fitted Model A and experimental loss data for M1924G material is presented in Figure 1.

![Figure 1: Error vs. induction at various frequencies (Model A)](image)

Model B: Allow the exponent of \( B \) to be different from 2 [3].

\[ P_{Fe} = k_hfB^4 + k_ef^2B^2. \]  

To use Model B, after dividing (5) by \( f \), apply a linear fit. Then use appropriate mathematical manipulations to obtain the model parameters (algorithm details can be found in [3]). The
model contains twelve unknown parameters. The accuracy of this model is presented in Figure 2.

![Figure 2: Error vs. induction at various frequencies (Model B)](image)

**Model C:** This model is a three term model similar to that proposed by Bertotti (1). It has been considered in [4]. In Model C, the hysteresis exponent is fitted to a third order polynomial as shown in Equation (6). The model is comprised of Equations (1), (6), (3) and (4). The algorithm for solving this model is relatively complex and the details can be found in [3]. One important additional equation required for solving this model is Equation (7), which is derived from the hysteresis loss component.

$$\alpha = \alpha_0 + \alpha_1 B + \alpha_2 B^2 + \alpha_3 B^3. \quad (6)$$

$$\log P_{Fe} = \log k_h + (\alpha_0 + \alpha_1 B + \alpha_2 B^2 + \alpha_3 B^3) \log B. \quad (7)$$

Model C contains thirteen unknown parameters and has been found to be applicable at low frequencies only (up to ~400 Hz). The results are shown in Figure 3.

![Figure 3: Error vs. induction at various frequencies (Model C)](image)

**Model D:** This two term model was proposed by [5] and is meant to provide better account of eddy current losses.

$$P_{Fe} = k_h B^d f + k_{eddy} B^2 f^2 (1 + d_3 B^{d_4}). \quad (8)$$

There are fourteen unknown parameters of the model and the algorithm for solving (8) is fairly complex. The results of this model are presented in Figure 4.

![Figure 4: Error vs. induction at various frequencies (Model D)](image)

**Model E:** The authors have proposed another variation of the Bertotti approach in [6]. The equations of the model are the same as that for Model C except Equation (6) is a quadratic polynomial. The algorithm, however is much simpler and the details can be found in [6]. The accuracy of this model is shown in Figure 5.

![Figure 5: Error vs. induction at various frequencies (Model E)](image)

### III. RESULTS AND REMARKS

A summary of the findings of the five models is presented in Table 1 in which the upper limit of the frequency against which the model was tested, the maximum error and the algorithmic complexity are summarized. Model E appears to give the best balanced result but has a higher number of unknown parameters compared to Model A. The results presented here should be seen as a first step towards the completion of a comprehensive review (in the longer version of this paper) that will include extensions to time domain as well as consideration of approaches for calculating iron losses for non-sinusoidal waveforms. We also view this work as setting the basis for including physical effects from manufacturing tolerances, stress and temperature effects and material anisotropy for iron loss prediction models in future.

<table>
<thead>
<tr>
<th>Frequency Range</th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
<th>Model D</th>
<th>Model E</th>
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<tbody>
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<td>~2 KHz</td>
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<td>~400 Hz</td>
<td>~2 KHz</td>
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<td>Complex</td>
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Table 1: Summary of various model performances

### References


