Calculation of eddy-current probe signal for a 3D defect using global series expansion

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Plan

1. Context
2. Challenges and existing solutions
3. Comparisons of the expansion functions
4. Illustrative examples
5. Conclusions
Context

- Eddy-Current Testing (ECT)
- Flaw reconstruction needs:
  - fast
  - reliable
  forward model + numerical solution
- Volumetric flaw model
- Integral equation scheme + Method of Moments
- **New basis functions for expansion**

The configuration:
- non-magnetic, conductive specimen
- time-harmonic excitation
- flaw = change of conductivity
- probe impedance variation at a set of probe positions
Classical challenges

- Excited region $\gg$ flaw volume
  $\rightarrow$ how to discretise the domain?

- Total probe impedance $Z$ $\gg$ variation of impedance $\Delta Z$
  $\rightarrow$ need for a direct formula for $\Delta Z$

$\downarrow$

integral equation models, $\mathbf{E} = \mathbf{E}^i + \mathbf{E}^d$
- “thin cracks”: surface integral equation;
- “volumetric flaws”: volume integral equation
- reciprocity theorem – formula for $\Delta Z$

- Numerical solution by the Method of Moments (MoM)
Today’s challenges

- Fast and reliable flaw characterisation
- Bad aspect ratio (flaw can be very thin)
- Volumetric model $\rightarrow$ surface model
- Optimisation-based inversion – sensitivity data
- Surrogate model of the ECT simulator – “smooth forward operator”
- ...
Thin crack model

Planar crack in the $x = 0$ plane $\rightarrow$ surface $\Gamma$ on which no current flows through

$$E_n\big|_\Gamma = 0 : E^i_x(r_0) + \lim_{r \to r_0} \left[ -j\omega\mu_0 \int_{\Gamma} G^{xx}(r, r') p(r') d\Gamma' \right] = 0; \quad r_0 \in \Gamma$$

Local approximation

![Local approximation diagram]

$$p(y, z) \approx \sum_{n=1}^{N} p_n \delta_n(y, z)$$

$$\delta_n(y, z) = \begin{cases} 
1 & \text{on the } n\text{th} \\
0 & \text{elsewhere} 
\end{cases}$$

$(Bowler, 1994.)$

Global approximation

![Global approximation diagram]

$$p(y, z) \approx \sum_{k=1}^{K} \sum_{l=1}^{L} p_{kl} v_{kl}(y, z)$$

$$v_{kl}(y, z) = \frac{\sin \frac{k\pi(y + B/2)}{B} \cos \frac{(2l - 1)\pi z}{D}}{\sin \frac{2\pi z}{D}}$$

$(Pávó & Lesselier, 2006.)$
Volumetric flaw model

Homogeneous conductivity $\sigma_0$ of the host locally changes to $\sigma(r)$ in the flaw $V$

$$
E(r) = E^i(r) - j\omega \mu_0 \int_V \mathcal{G}(r, r')P(r')dV';
$$

$$
P(r) = [\sigma(r) - \sigma_0]E(r)
$$

Local approximation

$$
P(x, y, z) \approx \sum_{n=1}^{N} P_n \delta_n(x, y, z)
$$

$$
\delta_n(x, y, z) = \begin{cases} 
1 & \text{in the } n\text{th} \cr 
0 & \text{elsewhere} \end{cases}
$$

(Bowler et al., 1991.)

Proposed herein, next slide...
The volumetric model with global expansion functions

\[ \mathbf{P}(x, y, z) \approx \sum_{k=-K}^{K} \sum_{l=-L}^{L} \sum_{m=-M}^{M} \mathbf{P}_{klm} v_{klm}(x, y, z) \]

- Globally defined \( v_{klm}(x, y, z) \) expansion functions
- Orthogonal function set, our choice is the Fourier-basis:
  \[ v_{klm}(x, y, z) = \exp \left[ 2\pi j \left( \frac{kx}{A} + \frac{ly}{B} + \frac{mz}{D} \right) \right] \]
- Other choices (e.g., Legendre or Chebyshev polynomials) are also possible
- Smooth approximation of \( \mathbf{P}(x, y, z) \)
- Many more advantages are expected...
Comparison of the local and global approximations

1. Convergence w.r.t. the discretisation

- **Local approximation, refinement**: $N_1 < N_2$:

  \[ P^{(1)}(x, y, z) \approx \sum_{n=1}^{N_1} P_n^{(1)} \delta_n^{(1)}(x, y, z) \]

  \[ P^{(2)}(x, y, z) \approx \sum_{n=1}^{N_2} P_n^{(2)} \delta_n^{(2)}(x, y, z) \]

  All basis functions change.

- **Global approximation, refinement**: $K_1 < K_2$, $L_1 < L_2$, $M_1 < M_2$:

  \[ P^{(1)}(x, y, z) \approx \sum_{k=-K_1}^{K_1} \sum_{l=-L_1}^{L_1} \sum_{m=-M_1}^{M_1} P_{klm}^{(1)} \psi_{klm}(x, y, z) \]

  \[ P^{(2)}(x, y, z) \approx \sum_{k=-K_2}^{K_2} \sum_{l=-L_2}^{L_2} \sum_{m=-M_2}^{M_2} P_{klm}^{(2)} \psi_{klm}(x, y, z) \]

  Basis functions from the previous stage can still be used, too.

  Higher coefficients are decreasing $\Rightarrow$ convergence.
2. Flaw reconstruction via minimizing an objective function

Change in the defect size:

- Local approx.:
  - speeding-up by pre-calculated matrix elements
  - fixed cell-size → only discrete flaw sizes

- Global approx.:
  - all matrix elements have to be re-computed
  - the objective function is expected to be smooth
  - the same discretisation can be used for different flaw sizes
  - sensitivity is expected to be easier to compute numerically
Comparison of the local and global approximations

3. Computation of the matrix elements
(integrals of the Green’s function)

For planar specimens: analytical expressions exist for both...

- **Local:** \( \int \delta_{nt}(r) \left( \int_{z'} G(r, r') \delta_{ns}(r') \, \text{d}r' \right) \, \text{d}r \)
  - small, localised sources
  - field is sharp in the spatial domain, spectrally extensive

- **Global:** \( \int \sum_{k} v_{k t} l_{m t} (r) \left( \int_{z'} G(r, r') v_{k s} l_{m s} (r') \, \text{d}r' \right) \, \text{d}r \)
  - smooth (space-harmonic), extensive sources
  - field is smooth in the spatial domain, spectrally narrow

4. Restrictions for the flaw shape

- **Local:** arbitrary shapes can be assembled
- **Global:** must be rectangular parallelepiped
Illustrative examples

Zero-conductivity flaws (voids) are considered.
Surface and volumetric models vs. measured data #1

JSAEM OD–60 Benchmark 150 kHz

Flaw:
- length = 10 mm
- width = 0.21 mm
- depth = 0.5 mm

Legend:
maximal harmonic orders are shown
- volumetric KLM
- surface KL
Surface and volumetric models vs. measured data #2

Flaw:
- length = 10 mm
- width = 0.21 mm
- depth = 0.75 mm

Legend:
- maximal harmonic orders are shown
- volumetric KLM
- surface KL
Flaw:
- length = 12.6 mm
- width = 0.28 mm
- depth = 5 mm

Legend:
- maximal harmonic orders are shown
- volumetric KLM
- surface KL
Convergence w.r.t. the harmonics order

$\Delta Z_{KLM}(y_c)$ : impedance variation on the line $y_c = 0 \ldots 10\text{ mm}$
computed by using maximal harmonic orders $K$, $L$ and $M$.

$\Delta Z_{121}(y_c)$ : the reference signal

$\| \cdot \|$ : $L_2$ function norm

![Graph showing impedance convergence](image-url)
Conclusions

- ECT modeling problems are still challenging
- Volume integral method have been studied for decades
- New discretisation scheme: expansion by harmonic basis functions
  - Control over the convergence
  - Less sensitive to bad aspect ratios
  - Small change of flaw size → small change of signal
  - Smoother basis functions → well-behaved fields
  - Restrictions for the shape
- Good agreement with measurements even with low-order series
- Many more parametric studies and comparisons are needed
  (??? → commercial use ???)
